THE MEASURE OF BEAUTY AND THE BEAUTY OF MEASURE

Abstract: The intersection of ancient art and mathematics was explored in a pair of courses taught together in a First Year Program at Randolph-Macon College. While the art history was a standard survey from Paleolithic to Renaissance, the focus was on the co-development of art and mathematics as indicators of cultural identity and achievement. Topics such as the understanding of canons and metrological systems were explored in conjunction with the rise of numerical systems, the functional applications of geometry and the ancient understanding of rational (commensurate) and irrational (incommensurate) numbers such as π *and* φ*.*

In 2005–2006, Randolph-Macon College instituted a new curriculum, with a centerpiece First-Year Experience. The FYE links several components of freshman education into one year-long integrated program. The components include writing, rhetoric, research skills and synthesis of disciplines. This is not a "multi-discipline" or "interdisciplinary" course, but instead allows for a close pairing of two courses in different disciplines but focusing on one theme or problem. Some links, such as literature and psychology focusing on the social and personal roles and benefits of laughter, or environmental science and political science focusing on water use, zoning and public policy, seem obvious and natural.[1]

But Math and Ancient Art History? Perhaps not so obvious. The courses were taught by classicist Elizabeth Fisher and mathematician John Rabung from the Department of Computer Science. We called the courses "The Beauty of Measure" and "The Measure of Beauty." We taught 30 students over the academic year: each of us had 15 students in our own course for Fall semester, then switched places in Spring term, with several hours of joint lectures, workshops and co-curricular events shared by the two classes throughout the year. We also sat in on one another's courses as much as possible, so that we could fully integrate content, assignments and objectives. The math course fulfilled a math requirement. The art history course was the equivalent of the first half of an art history survey, the typical "Grotto to Giotto" sequence.

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[1] The courses discussed in this article were part of one FYC in the third year of a pilot program for the new curriculum; see Appendix, below.

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Mathematics and Art as Communication

We approached our joint courses with the basic idea that math and art share a purpose of communication through languages and patterns of thought that differ from spoken or written languages. These universal languages in some ways transcend the incompatibilities of spoken and written languages, although just as one must learn foreign languages, one must also learn to read and understand the languages of math and art.

The math course and the art history survey both began with the Paleolithic. Both classes read a chapter from Stephen Jay Gould's *Leonardo's Mountain of Clams and the Diet of Worms*, which debunks the notion of cave art as "primitive."[2] The math class considered the definitions of number and counting, seemingly simple to us through familiarity, but in fact profound for our understanding of oneness, plurality and order. Of particular interest as possible examples of early counting, calendars or enumerative sequences were the Ishango bone,[3] the Blombos Cave bone[4] and notched bones from Paleolithic contexts in Europe.[5] The art class considered the earliest manifestations of human art: portable sculptures and parietal art. How do we read the images of Chauvet and Lascaux Caves? Can it be mere coincidence that notched bones, musical instruments, sculptural and parietal art all appear in the human record at approximately the same time?[6]

Numbers and Writing in Mesopotamia

This was a survey course, so we moved on quickly to Mesopotamia. Our next topic was the genesis of writing and accounting as postulated by Schmandt-Besserat on the basis of the 1000s of clay tokens used throughout southwest Asia during the Neolithic period.[7] In Mesopotamia, measurement became the way of organizing the world. We looked at the practicality of the Sumerians and Babylonians; art, mathematics and writing were tools which were arguably requisite to the birth of civilization, as manifest in monuments, cities, laws and literature. Highlights of the art history class included the Gudea statues, some of which portray the ruler Gudea with architectural plans and rules inscribed on his cloak, and the use of propor-

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[2] Gould (1998) 161–78.

[3] Bogoshi, Naidoo and Webb (1987) 294.

[4] Henshilwood (1997) 890–5.

[5] Cf. d'Errico and Henshilwood (2007) 142–63.

[6] For a pertinent study, see d'Errico (2003) 1–70.

[7] Schmandt-Besserat (1995). We found Sasson (1995), in which this article appears, particularly accessible to the freshmen in this course. Other articles from the volume which we used, each with a bibliography for further research, were Azarpay (1995); Powell (1995); Rochberg (1995); and Winter (1995).

tions and scale to communicate status in relief sculpture.[8] The math class looked at the sexagesimal system, the foundation for our 360-degree circle and 60-minute hour. Advanced mathematics, though difficult to interpret, are evident in documents such as Plimpton 322, a cuneiform tablet from Babylonia dating to 1800 BC.[9] The use of a different numerical base and ancient arithmetic procedures made the students focus more closely on solutions to apparently familiar problems, for example these quadratic equation exercises done in Babylonian math:

1. (From an old-Babylonian text BM 85 196, problem no. 9.) A beam of length 0;30 stands (vertically) against a wall. The upper end has slipped down a distance 0;6. How far did the other end move?
2. (A problem similar to the one found in YBC 6967 and discussed in class.) A rectangular field has an area of 3600 square yards. Its length exceeds its width by 35 yards.
    a. Play the role of a Babylonian finding the length and width of the field by writing and following exactly the practical recipe given in Robson's article (Figure 10, p. 23).
    b. Now do the problem using the algebra techniques you learned in high school.

Egyptian Geometry in Mathematics and Art

The Egyptians had to contend with the yearly flood of the Nile plain. Herodotus relates that the Pharaoh Sesostris divided up the land into square plots of equal size, which were assigned to individuals to be worked and taxed. When the floods carried away a portion of the land, the Pharaoh sent examiners to re-measure the plot and reassess the tax burden. Thus, Herodotus concludes, δοκέει δέ μοι ἐνθεῦτεν γεωμετρίη εὕρεθεῖσα ἐς τὴν Ἑλλάδα ἐπανελθεῖν (2.109.3). How fitting that among the monuments of ancient Egypt are the supremely geometric shapes of the pyramids, axially arranged hypostyle halls, and a perspective in art that renders individually the elements of a landscape, as in the drawing of the pool and garden from a New Kingdom Tomb,[10] or the limbs and features of a person, as seen on an artist's practice board, both now in the British Museum,

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[8] Amiet (1977) 378, 448. E.g., this statue of Gudea in the Louvre is from Girsu, c. 2150 BC, and shows Gudea as architect with plan and rule engraved on his lap. See also Aruz (2003) 426–7.

[9] Neugebauer (1969) 29–52; Buck (1980) 335–45; Robson (2002) 105–20.

[10] A garden pool: fragment of wall painting from the tomb of Nebamun (no. 9), Thebes, Egypt, 18th Dynasty, c. 1350 BC, British Museum, Salt Collection, EA 37983. www.britishmuseum.org, key word: Nebamun.

to form a whole that is literally the sum of its parts! Egyptian mathematics also relied on unit fractions, breaking larger units into smaller and smaller units to organize and count and divide and put back together into a different whole. Math problem sets were drawn in part from ancient Egyptian math problems found on the Rhind Mathematical Papyrus.[11] An example:

1. Use the Egyptian multiplication technique of repeated doubling to show the details of multiplying 37 by 46.
2. Use Ahmes' method (as outlined in the McLeish reading)[12] to compute $^{2}/_{3}$ of $^{1}/_{23}$. Show that your answer is correct by doing a modern addition of the two unit fractions.
3. Ahmes' table gives 2:17 to be $/_{12} + /_{51} + /_{68}$. Use modern fraction arithmetic to verify that the sum of the three unit fractions really is $^{2}/_{17}$.
    a. Given Ahmes' expression for 2:17, what would he give for 4:17? Use our doubling technique to easily get the answer.
    b. Knowing from part a. the unit fraction expression for 4:17, what would be the expression for 8:17? Again double 4:17, but note that you may need Ahmes' table this time.
    c. By doing Egyptian multiplication (successive doubling) compute 13 times $^{1}/_{17}$.
4. Try your algebra skills on this one. At several places in the Rhind Papyrus, Ahmes indicates, in effect, that the area of a circle is found by squaring $^{8}/_{9}$ of its diameter. Given our modern-day formula for computing the area of a circle (π times the square of the radius), what value were the Egyptians (unwittingly?) using for π? What value do we assign to π?

Another interesting Egyptian math problem is computing the volume of a truncated pyramid, or frustrum, a problem presented on the Moscow Mathematical Papyrus.[13] Many claims have been made in the past about the mathematics evident in the construction of the pyramids, although the Old Kingdom pyramids are much older than any of the math papyri. While claims of Egyptian use of π are generally met with skepticism, the pyramids were built with a unit known

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[11] The papyrus is a Second Intermediate Period copy of an earlier mathematical teaching text originally dating from perhaps 1650 BC. The scribe who made the copy is Ahmes. The papyrus is in the British Museum, pBM 10058.

[12] McLeish (1991) 39–52.

[13] Gillings (1982) 187.

as a *seked*, which is the ratio of the inclination of one side of the pyramid to the horizontal base, calculated as a lateral displacement of 7 palms per drop of one royal cubit. Thus the lower the *seked*, the steeper the slope. The *seked* of the "Great Pyramid" at Giza is 5½, while the pyramid of Chephren at Giza has a *seked* of 5¼. The Bent Pyramid of Dahshur has a *seked* of 5 at the lower face, with a *seked* of 7½ on the upper face. The debate as to whether the Egyptians used the irrational number π is continued in regard to φ, an irrational number also known as the Golden Ratio, about which more is said below.

Systems of Exchange in the Bronze Age

For the Aegean Bronze Age cultures, we focused on the systems of exchange between the Aegean, Egypt and Mesopotamia, which required concepts of worth, price and value. The cow was a common unit of value, reminding us of Eurycleia in the *Odyssey*, whom Laertes purchased for the apparently extravagant price of 20 cows.[14] The ship wrecked at Uluburun carried oxhide ingots, a Bronze Age bullion of copper ore with a very high value.[15] Value is expressed in the proportional weights used to facilitate exchange between Egypt, the Aegean and Mesopotamia.[16] The importance of this exchange was communicated through the artistic enterprise, chiefly with specialized forms of art dedicated to the gods and consumed by the rulers and other elite members of these societies. The art course highlighted the use of the double pan balance as a symbol of exchange, as well as a metaphor for justice, as in the Egyptian *Book of the Dead*[17] or the weighing of the fate of Achilles.[18]

Commensurable and Incommensurable in Greek Mathematics and Art

The Greek math lessons focused on the nature of proof. Pythagorus' harmonies, Euclid's mousetrap proof of the Pythagorean theorem and Archimedes' recurrence equation were all manifestations of Greek skepticism. Particular emphasis was given to the meanings of commensurable and incommensurable, and to proofs *ad absurdum.* The math exercises brought into focus the importance of inquiry in ancient Greece, while providing a chance to re-examine high school algebra and geometry at a higher level.

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[14] *Od.* 1.430–4.

[15] Pulak and Bass, http://ina.tamu.edu/ub_main.htm. Accessed 11/05/07. Includes an excellent bibliography.

[16] Petruso (1992) 15–19, 69–75.

[17] Called *psychostasis;* see Budge (1898) xciv, 12; Faulkner (1993) 14.

[18] *Il.* 22.179.

Reviewing Some Archimedean Techniques

Recall that we (and our textbook)[19] gave the idea of the proof of Archimedes' law of the lever for commensurable weights:

> Commensurable weights balance at distances from the fulcrum that are inversely proportional to their magnitudes. That is, if commensurable weights W and w are at distances D and d, respectively, from the fulcrum, then $D/d = w/W$. (Equivalently, $WD = wd$, but Archimedes would not have multiplied a weight by a distance.)

The idea that we gave assumed our commensurable weights, W and w, had a ratio of 4/3; that is, there is a common measure, say m, of W and w such that $W = 4m$ and $w = 3m$. We reasoned from there that the distances at which these two weights balance, D and d respectively, are such that $D/d = 3/4$. To show that you understand this proof idea (Stein shows the same development, but with ratio 5/3), present it assuming that $W/w = 7/4$. Write your proof as a good English paragraph with sound logic and accompany it with diagrams illustrating your reasoning.

1. Having established the law of the lever, we should now be able to solve some simple balancing problems:
    a. If a weight of 36 pounds is placed on a balance arm 8 feet from the fulcrum, how far from the fulcrum on the other side of the balance arm should a weight of 24 pounds be placed in order to bring the system into perfect balance?
    b. If a weight of 250 pounds is placed on a balance arm 15 feet from the fulcrum, how much weight would we have to place at a position 18 feet from the fulcrum on the other side of the balance arm in order to bring the system into perfect balance?
    c. On the left side of a balance arm weights of 5 and 8 pounds are placed 12 and 20 feet, respectively, from the fulcrum. On the right side of the balance arm, how far from the fulcrum should a 20 pound weight be placed in order to bring the system into perfect balance? (*Hint: Find the center of gravity of the weights on the left and then consider all the weight on the left to be concentrated at that center of gravity*.)

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[19] Stein (1999) 7–14.

Greek art provides many examples of the use of mathematics. One student exercise was to draw the Doric temple as described by Vitruvius.[20] We read Pliny on the Canon of Polykleitos and discussed the relative proportions of 5th- and 4th-century Greek sculptures. We also studied images of a block from Salamis carved as a standard for linear measures including the foot, rule, hand-span, half of an arm-span, and a cubit, and another in Oxford with a full span.[21] These metrological reliefs offer a different, physical interpretation of the well-known claim that "Man is the measure of all things."

The Apollo Temple at Didyma also offers an example of mathematical planning in Greek architecture. On the inner walls of the temple are inscribed lines providing a scale of the building's elements: capitals, columns and architraves, all carefully laid out by the architect/designer for the contractors to follow. If the temple had been finished, these lines would have been chiseled off in a final polishing. Luckily for us, the people of Didyma ran out of money. The lines left on the unfinished walls offer an example of the mathematical ratios set out by an architect for the builders to follow.[22]

The Invention of Linear Perspective in Art

In Roman and later Renaissance art, we considered the development of linear perspective. Most scholars say that linear perspective was not invented until the Renaissance.[23] We compared panels of Roman and Pompeian frescoes with Renaissance works, and found that while linear perspective was not clearly present as an overall scheme for entire walls in Roman houses, individual panels did seem to maintain reasonably consistent one-point perspective. One lively class discussion had students take opposing sides: one group argued that true linear perspective was known and that the preserved paintings were composed with single vanishing point construction, even if the execution was imperfect, while the other argued that the heavy reliance on atmospheric perspective, combined with a lack of mathematical treatises, meant that Roman artists approximated linear perspective intuitively. To further explore the use of linear perspective in two-dimensional art, we analyzed engravings and woodcuts by Albrecht Dürer, in particular the drawings accompanying his 1525 work entitled *Underweysung der Messung mit Zirckel und Richtscheyt in Linien* ("Instructions in Measuring with Compass and Straight Edge in Lines").[24]

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[20] Vitr. *de Arch.* 3.1.1–3.4.5.

[21] Dekoulakou-Sideris (1990) 445–51.

[22] For a summary in English, see Haselberger (1985).

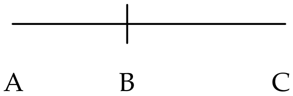
[23] Field (1997).

[24] Field (1997) 121.

The Golden Ratio

Probably the most fascinating and *au courant* math came as we studied the irrational number φ: the Golden Ratio. Our co-curricular events for the year included lectures by the authors of books about the Golden Ratio: Bülent Atalay of the University of Mary Washington and Mario Livio from Johns Hopkins University. These two scholars presented different points of view about the presence of the irrational number 1.61803 in art with mind-teasing puzzles and examples.

A problem, known in ancient Greece,[25] sets forth the ratio of φ — and it seems so simple. Can you divide a line of any length into two segments, so that the ratio of the smaller segment to the larger segment is the same as the ratio of the larger segment to the whole line?



AB/BC as BC/AC

Euclid referred to this line as "cut in extreme and mean ratio." Mathematically, the ratio can be calculated as a constant, which is the irrational number φ, or 1.618… . Many claims have been made about this number. Is φ the number of beauty? Is it coded into the Pyramid of Khufu, the Parthenon façade and the Mona Lisa? Did you know that the Golden ratio shows up in the kernels of a pine cone and the spiral of a nautilus and the petals of a rose?

The students particularly enjoyed the puzzle of φ. They discussed the famous Rabbit problem set out by the Italian Renaissance mathematician Leonardo of Pisa, also known as Fibonacci, c. 1202 CE.[26] While there have been numerous claims about the mystical and profound aspects of the number φ,[27] the first treatise on the aesthetics of the Golden Ratio appeared in the work of Luca Pacioli, *De Divina Proportione*, published in 1509. Pacioli's work influenced Leonardo da Vinci (1452–1519) and Leon Battista Alberti, whose 1452 volume on architecture, *De re aedificatoria,* was patterned on Vitruvius, leading us back to the ancient Romans.

From Paleolithic notched bones to the irrational number φ, the intersection of math and art provided a new way to engage students in looking at both disciplines. The chief advantage to the joint courses for the study of antiquity was the sense the students took away that

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[25] Euclid's Proposition VI, 30.

[26] Livio (2002) 96–109 has an explanation of the Fibonacci Series. The cover of the paperback edition of Livio's book has a quote from Dan Brown, author of the popular novel *The Da Vinci Code*, which featured the Fibonacci series as the "Code."

[27] For instance, see Lawlor (1982) for a variety of references to Greek, Hindu, Chinese and Egyptian philosophy.

the intellectual achievements of ancient cultures were complex, interesting, experimental and fundamental to our own times. The math served to "modernize" antiquity in a way rarely grasped by students looking at broken statues and column stumps. Further, by discussing the meaning of math/art terms such as proportions, balance, symmetry and perspective, students gained an understanding of the complex philosophical natures of both disciplines. And by looking for dimensions, scale, plan and design in architecture and art, they were drawn to look more closely at ancient monuments and art, and perhaps came to see them as remarkable displays of genius and skill that would deserve study in any context.

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Appendix: First Year Experience

A third course for writing completes the First Year Experience. The curriculum page at www.rmc.edu describes the FYC as follows:

First-Year Experience The three-course First-Year Experience (FYE) is required of all first-year students for full implementation beginning with the 2005–06 academic year. The FYE will consist of: **1)** A First-Year Colloquium (FYC) that will

- be taught by at least two full-time faculty members from different disciplines (departments),
- take place over two terms beginning with the Fall term,
- carry 3 or 4 credits each term (based on the instructor's recommendation),
- focus on engaging issues that in some meaningful way cross disciplinary boundaries,
- meet Areas of Knowledge requirements in appropriate disciplines,
- include activities and tasks that strengthen student skills in writing, reading, speaking, listening, and the use of information resources,
- incorporate a substantive project that engages the students in actively exploring the cross-disciplinary aspects of the colloquium,
- infuse active learning into its pedagogy, and include co-curricular activities as appropriate to the instructor's topics/issues and discipline....